Nuances
Estudos sobre Educação

# THE TEACHING OF GEOMETRY BASED ON THE SYSTEM OF GEOMETRIC CONCEPTS

## *O ENSINO DE GEOMETRIA COM BASE NO SISTEMA DE CONCEITOS GEOMÉTRICOS*

## *LA ENSEÑANZA DE LA GEOMETRÍA BASADA EN EL SISTEMA DE CONCEPTOS GEOMÉTRICOS*

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**ABSTRACT**: This article is the result of research carried out in the doctorate, which had as its object the organization of geometry teaching in the early years of schooling to seek theoretical-methodological subsidies to overcome the concept of teaching presence in the practices of teachers who teach in this teaching stage, in which the practical aspects of geometric concepts are predominantly emphasized. In this text, we focus on revealing the essential unit for teaching geometry, the concept's meaning, and theoretical generalizations as premises for forming the system of ideas. For that, we used as an academic and methodological basis the principles of the Historical-Cultural Theory and the assumptions of the Teaching Guiding Activity, through which they revealed that the essential unit for the systematization of geometric concepts refers to the control of variations in the dimensions of objects and spaces. We conclude by stating that each dimension (length, width, height/depth) represents a magnitude that triggers the relations of commonality and generality between the inferior concepts that, when generalized in a hierarchically distinct category from the others, become superior, contributing to the development of foundations of students' theoretical thinking.

**KEYWORDS**: Historical-Cultural Theory. Teaching Guiding Activity. System of geometric concepts.

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***RESUMO***: *O presente artigo é resultado da investigação realizada no doutorado, que teve como objeto a organização do ensino de geometria nos anos iniciais de escolarização, com o intuito buscarmos subsídios teóricos-metodológicos com vistas a superar a concepção de ensino presente nas práticas dos professores que lecionam nesta etapa de ensino, em que se enfatiza, predominante, os aspectos empíricos dos conceitos geométricos. Neste texto, focalizamos em revelar a unidade essencial para o ensino de geometria, o significado de conceito e as generalizações teóricas como premissas para a formação do sistema de conceitos. Para tanto, utilizamos como base teórica e metodológica os princípios da Teoria Histórico-Cultural e os pressupostos da Atividade Orientadora de Ensino, por meio dos quais desvelaram que a unidade essencial para a sistematização dos conceitos geométricos se refere ao controle das variações das dimensões dos objetos e espaços. Concluímos afirmando que cada dimensão (comprimento, largura, altura/profundidade) representa uma grandeza que desencadeia as relações de comunalidade e generalidade entre os conceitos inferiores que, quando generalizados em uma categoria hierarquicamente distinta das demais, se tornam superiores, contribuindo para o desenvolvimento das bases do pensamento teórico dos estudantes.*

***PALAVRAS-CHAVE***: *Teoria Histórico-Cultural. Atividade Orientadora de Ensino. Sistema de conceitos geométricos.*

***RESUMEN***: *Este artículo es resultado de una investigación realizada en el doctorado, que tuvo como objeto la organización de la enseñanza de la geometría en los primeros años de escolaridad, con el fin de buscar subsidios teórico-metodológicos con el fin de superar el concepto de enseñanza presente en las prácticas del profesorado que imparte docencia en esta etapa docente, en la que predomina el énfasis en los aspectos empíricos de los conceptos geométricos. En este texto nos enfocamos en develar la unidad esencial para la enseñanza de la geometría, el significado de concepto y las generalizaciones teóricas como premisas para la formación del sistema de conceptos. Para ello, se utilizó como base teórica y metodológica los principios de la Teoría Histórico-Cultural y los presupuestos de la Actividad Orientadora Docente, a través de los cuales se reveló que la unidad esencial para la sistematización de los conceptos geométricos se refiere al control de las variaciones en las dimensiones. de objetos y espacios. Concluimos afirmando que cada dimensión (largo, ancho, alto/profundidad) representa una magnitud que desencadena las relaciones de comunalidad y generalidad entre los conceptos inferiores que, al generalizarse en una categoría jerárquicamente distinta de los demás, se vuelven superiores, contribuyendo al desarrollo de fundamentos del pensamiento teórico de los estudiantes.*

***PALABRAS-CLAVE***: *Teoría Histórico-Cultural. Actividad de Orientación Docente. Sistema de conceptos geométricos.*

**Introduction**

The search for overcoming the conception of mathematics teaching in the early years of schooling, especially geometry present in educational institutions, was the object of study of several authors, among which we highlight the investigations conducted by Locatelli (2015), Locatelli and Moraes (2016), Ferreira (2017), Assumpção (2018), Santos (2020), and Ferreira and Moraes (2021), whose analyses pointed out that mathematics tasks remain in the appearance of concepts, assuming a technical and repetitive direction, given that these are worked in isolation from the social determinants that surround them.

However, this form of teaching restricts the students' psychic development, which is analyzed in terms of quality in the face of the capitalist society we aspire to confront. In this educational scenario, we defend the unity of concepts instead of their fragmentation, the appropriation instead of the repetition of techniques triggered by the fragmented work performed from isolated elements, that is, we assume an education based on the system of concepts aiming at the formation of the theoretician that opposes to the thought organized by the sensorial perception of the apparent.

Davídov's studies (1988, p. 6, our translation) reveal that theoretical thinking consists of "[...] a particular procedure with which man focuses on the understanding of things and events through the analysis of the conditions of their origin and development. In this sense, the question arises: How to organize teaching based on the system of geometric concepts with a view to the formation of theoretical thinking?

It is necessary to study how the subject learns and the essential unity of geometric concepts so that, later, teaching proposals can be systematized for this area of knowledge in the early years of schooling.

To this end, the present text is organized into four parts: first, the presentation of the assumptions of our theoretical support that underlie teaching organization, especially geometry. Then, in the second moment, we present the results of a synthesis of how the concept and the formation of a system of concepts represent the appropriate way for the appropriation of knowledge by students.

Next, we discuss the relationship between magnitudes as the general foundation for teaching mathematics, seeking to refine it for our object, arriving at its fundamental unit: the control of the variation of the dimensions of objects and spaces. Through this unit, we unveiled a system of geometric concepts that will add, to the collective effort of many, teaching that enables the appropriation of mathematical concepts, especially geometric ones, and the formation of theoretical thinking.

## Guiding Teaching Activity as a Theoretical and Methodological basis

In this research, we use the principles synthesized by Ferreira (2017) of the Cultural-Historical Theory and the assumptions of the Guiding Teaching Activity (AOE) as a theoretical and methodological basis that will subsidize the essential didactic and pedagogical elements that underlie the organization of teaching as an activity. According to Moura and Araujo (2020), the perception of this movement was evidenced more than 30 years ago, initially by Moura (1992) and Lanner de Moura (1995), when the teaching activity was first publicized as Guiding Teaching Activity (Guiding Teaching Activity). It is worth noting that including the term "guiding" in the concept of activity **reinforces intentionality in the development of the teaching activity.**

> Thus, the AOE concept's origin lies in teaching organization. But what kind of teaching? Teaching provides conditions for subjects to appropriate scientific knowledge and develop their intellectual capacities. At this moment of the concept's production, the guiding activity materialized in actions and instruments that the teacher could use to teach certain school content. Even in the initial consolidation process of the guiding activity, some understood it as the "learning triggering problem". However, it is essential to consider that in its germ, it already carried a structure that went beyond these elements, it revealed the most generalized direction for the organization of teaching since it contemplated the essential triad of pedagogical activity: content to be taught, the most appropriate form and the subjects-learners (CEDRO; MORETTI; MORAES, 2019, p. 433-434, our translation).

Currently, the Study and Research Group on Pedagogical Activity (GEPAPe) of the University of São Paulo (USP), led by Professor Dr. Manoel Oriosvaldo de Moura and Professor Dr. Elaine Sampaio Araújo, has continued the development of the concept of AOE from the conception of the organization of teaching as activity.

The investigations of Moura *et al.* (2010) pointed to the concept of activity - systematized by Leontiev (1978) - as the way to locate the essential didactic and pedagogical elements to support the teacher's work when organizing teaching. Thus, AOE will provide theoretical and methodological subsidies to overcome the dichotomy between theory and practice "[...] so that the educational process is constituted as an activity for the student and the teacher" (MOURA *et al.,* 2010, p. 96, our translation).

> AOE maintains the activity structure proposed by Leontiev by indicating a need (appropriation of culture), a real reason (appropriation of historically accumulated knowledge), objectives (teaching and learning), and proposing actions that consider the objective conditions of the school institution (MOURA *et al.,* 2010, p. 96, our translation).

Since AOE seeks the concept of activity and its structuring elements (necessity, reason, actions, and operations), it assumes the quality of mediation, given that the activity is the unit of life mediated by the psychological reflex, which guides the subject in the objective world (LEONTIEV, 1978). In the words of Moura *et al.* (2010, p. 97, our translation), AOE

> [...] as a process of approaching the object: the knowledge of new quality [...] takes the dimension of mediation when constituting itself as a mode of realization of teaching and learning of the subjects that, when acting in a learning space, are modified and, thus, will also constitute themselves in subjects of new quality.

AOE takes the guiding and executing dimension in organizing and developing teaching and learning. For Araújo (2019), the need, the motive, and the object of AOE constitute its guiding dimension.

> In general terms, AOE's motive is to enable the social experience of humanity, objectified in culture, to become the subject's experience, such that the object of AOE is the historically produced theoretical knowledge. However, what relates the motive to the object, in this perspective, is the social need for the formation of the human personality [...] (ARAUJO, 2019, p. 132-133, our translation).

In the execution dimension,

> [...] entram em cena as ações e operações para que o motivo se realize no objeto. As ações, em termos gerais, estão voltadas a objetivos específicos que, no caso da educação escolar, se identificam com a apropriação dos conceitos científicos; e as operações, para que essas ações se efetivem, passam por modos de ação que desenvolvam o pensamento teórico (ARAUJO, 2019, p. 133, our translation).

The activity is always linked to a need, to the man's needy state of something that presents an objective character, because the object of the action, whether internal or external, refers to what gives it a particular orientation, which is related to the practical motive of the activity (LEONTIEV, 1978). For this reason, the educational process must provide those who teach and those who learn with authentic and meaningful motives for teaching and learning certain school content.

From this point of view, the studies by Moura *et al.* (2010, p. 220, our translation) reveal that "The quality of activity to teaching is given by the need to provide the appropriation of culture that can mobilize subjects to act towards the achievement of a common goal. In this case, the teacher in the teaching activity tries to organize it properly so that the student can appropriate the historically systematized culture and develop it.

Thus, we aim to unveil the organization of geometry teaching based on the system of concepts to develop students' theoretical thinking in the early elementary school years.

## Concepts and system of concepts

Before going into the concepts and the concept system, we cannot mention that their understanding results from the investigative analysis path that Vygotsky (2001) advocated on the relationship between thought and language. Unlike the investigative methods developed in his time, which considered the studies of psychological functions separately as processes isolated from the whole, Vygotsky (2001) uses the analysis based on the study by units, not elements. "When we speak of the unit, we refer to a product of analysis which, unlike that of elements, retains all the basic properties of the whole and cannot be divided without losing them." (VIGOTSKI, 2001, p. 25, our translation). Thus, when analyzed separately, language and thought cause the characteristics resulting from the connection between them to be lost; essentially, the relationship that constituted them is lost.

In the opposite movement, through what he called the unit of analysis, Vygotsky (2001) considers language as the main phenomenon of human thought. He supports this assertion based on his experimental research and theoretical analysis that reveals the word's meaning as the unit between the functions of language and thought (VIGOTSKI, 2001). For this reason,

> […] the method we must follow in our exploration of the nature of verbal thought is that of semantic analysis - the study of the development, functioning, and structure of this unit that contains interrelated thought and language (VIGOTSKI, 2001, p. 26, our translation).

The understanding of how semantic analysis presents itself in the formation and development of concepts and concept systems goes through the primary function of language, that is, communication, the rational and intentional transmission of experience and thought to the other, the social exchange originated in and through collective work. As we have already discussed, the means for this transmission is the sign (the word, the sound, the gestures, among others), moreover, for Vygotsky (2001, p. 27, our translation), "[...] true communication requires meaning, that is, both generalization and signs" that, later, become symbols.

To elucidate his point of view, the author uses the explanation of the German linguist Eduardo Sapir who discussed "[...] some of the problems and criticisms directed at the concept of culture" (GONÇALVES, 2021, p. 26, our translation), highlighting certain fundamental dimensions of sociocultural life, such as its dimension of individual experience:

> According to Eduardo Sapir's penetrating description, the world of experience can be significantly simplified and generalized before being translated into symbols. Communication is possible only in this way, for individual experience resides solely in one's consciousness and is, strictly speaking, incommunicable. To become communicable, it must be included in a specific category, which by tacit convention, human society regards as a unit. Thus, **true communication presupposes a generalizing attitude**, an advanced stage in developing the meaning of words. Higher forms of human exchange are possible only because man's thinking reflects a conceptualized reality (VIGOTSKI, 2001, p 27, our translation).

Considering the word's meaning as a unit encompassing both thought and social exchange, we can access a true (causal-genetic) analysis of its origin and development. As the quotation shows, we understand that an initially simple word, such as mother, for example, represents, in a simplified way, for the small child its experience with the one who feeds it, cleans it, puts it to sleep, takes care of it, etc., and which, in a generalized way, translates into the word mother.

As the child grows and develops, the word mother takes on new meanings, representing a caring, reproductive woman with her characteristics, etc. In the two exemplified moments, the mother presents different levels of generality, "[...] a typical combination of the concrete and the abstract" (VIGOTSKI, 2001, p. 152, our translation).

Now, it is worthwhile to refer to what Vygotsky (2001) analogically exemplified about the location of a concept in its highest abstraction and its objective place in reality. Based on the measurement of the globe from its geographic coordinates - the longitude and latitude - he explains that the first represents

> [...] the nature of the act of thought itself, of the very encompassing of objects in concepts, from the point of view of the unity of the concrete and the abstract contained therein. The latitude of the idea will first characterize the relation of the concept to the object, the point of application of the

> concept to a given point of reality (VIGOTSKI, 2001, p. 154, our translation).

From this point of view, we can understand that the word mother, in the first moment, refers to the point of application of the concept to reality, to latitude, that is, it is about the child's relationship with the mother in a practical, objective, direct way, as we mentioned: feed her, clean her, dress her, put her to sleep. The child could prolong this series because the new words are not complicated. But she has not yet assimilated the word "caregiver," which, consolidated in the longitude, is a more general concept that encompasses the words mentioned. Assimilating this new word means

> [...] to dominate the commonality relation, to acquire the first superior concept, which includes all the series of more particular ideas subordinated to it, to dominate a new form of movement of concepts not only in the horizontal plane [latitude], but also in the vertical plane [longitude] (VIGOTSKI, 2001, p. 154, our translation).

As in the example of the word mother, we see Vygotsky's (2001) conception that, initially, the word is a generalization of the most elementary kind. As the child develops, it moves from elementary generalization to higher and higher forms of generalization. For the author, this process culminates in forming authentic and accurate concepts (VIGOTSKI, 2001). Thus, we corroborate his categorical statement that

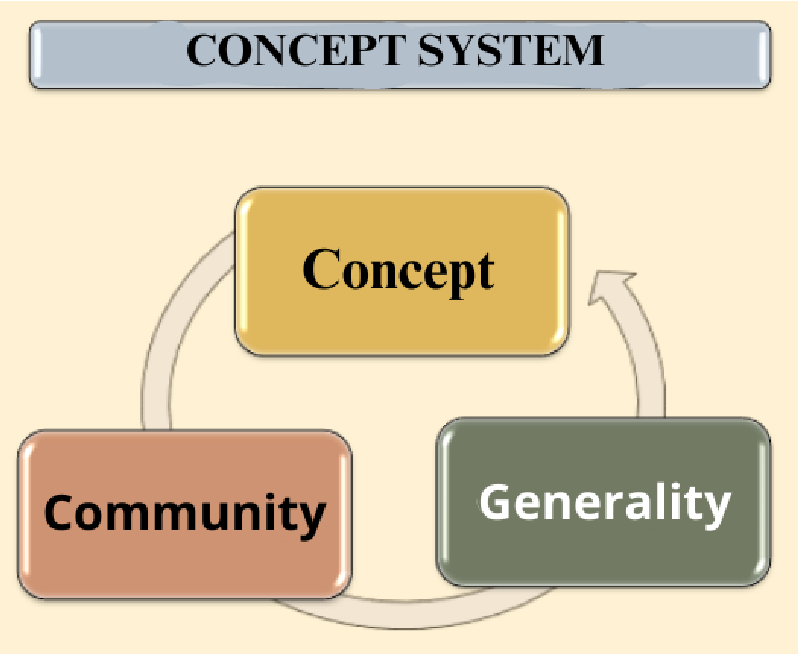
> […] the meaning of the word is nothing more than a generalization or a concept [...]. Generalization and word meaning are synonyms. Every concept's generalization and formation constitutes the most specific, authentic, and undoubted act of thought (VIGOTSKI, 2001, p. 289, our translation).

Given this, we consider that **the concept is the word's true meaning, a complex act of thought, a generalization linked to its causal-genetic development**.

Vygotsky's studies (2001, p. 2017, our translation) point out that a system of concepts is revealed as the "[...] **relations of concepts among themselves**". Thus, the system presents a dialectical relationship intertwined with the "dynamic-causal" and "genetic causal" nexuses that act as the link or connection between the concepts, forming a structure (system), in interdependence with the production - creation, and development - of the means to satisfy human needs.

The system of concepts is formed in the logical-historical movement of a given object content, in the dynamics of its structuring, development, and transformation of phenomena. In this sense, we aim to represent this system using the following figure:

**Figure 1 -** Subjects' formation of the concept system




Source: Elaborated by the authors

As we have pointed out, our investigations resorted to Vygotsky's studies (2001) to explain the formation of concepts, in which he showed that

> [...] the development of concepts begins with the relation of communality - perception of common traits between objects - constituting the most sensitive form between the meanings of words (concepts). Communality presents a close relationship with the structures of generalization (syncretic grouping, complex, prejudice, concept), which, in turn, are equivalent to the specific system of commonality, its measure of unity of the abstract and concrete (FERREIRA, 2017, p. 161, our translation).

Thus, the development process of concepts or word meanings occurs with the transition from one generalization structure to another, forming a system. However, it should be noted that concepts are not assimilated and directly. It is a complex act of thought because it goes through complex psychic processes or higher psychological functions (voluntary attention, logical memory, abstraction, etc.) directly linked to the subject's relationship with the world (VIGOTSKI, 2001).

Based on these assumptions, in the next topic, we will analyze the mathematics teaching object and the essential unity of geometric concepts to unveil its conceptual relationship system from the essential unity that engenders it, aiming at an adequate organization of geometry teaching that enables the appropriation and development of theoretical thinking by students.

## The relationship between magnitudes as the general foundation of mathematics teaching

One of the fundamental assumptions of the Guiding Teaching Activity (Guiding Teaching Activity) corresponds to the study of the object of the subject to be taught, that is, the investigation of the essence of the concept, its origin, and its general foundation. This principle should be considered in organizing the teacher's teaching activity to contain the interdependent relationship between the concepts: their unity. In this sense, we should analyze how the concept of magnitude contributes to the organization of mathematics teaching to reach the essential unity of our object.

For Davídov (1988, p. 208, our translation), the concept of magnitude is connected with the relations of equal, more significant, and lesser: "The multiplicity of any object becomes a magnitude when criteria are established that allow us to determine whether A is similar to B, more significant than B, or less than B. For Moura *et al.* (2018, p. 5-6, emphasis added, our translation),

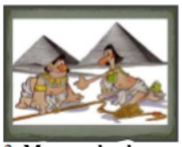
> [...] **magnitude** can be defined as a quality of an object or phenomenon that can be **quantified**. A quality of an object or phenomenon can be understood as the set of relations we establish between these objects or phenomena. Thus, the perception of quality is always relative to something, resulting from comparison and identification. The **quality** can admit a variation according to a quantity, which can be translated into numbers. The **courage** quality of a person, for example, admits an interpretation according to quantity, but this variation is not translatable into numbers, therefore, it cannot be measured. For example, it makes sense to say that John is braver than Anthony, but not that John's courage is twice as much as Anthony's.

Under this logic, Moura *et al.* (2018, p. 6, our translation) state that to express the qualities of an object or phenomenon in numerical form, it is necessary to perform three steps:

> Find another object or phenomenon with the same magnitude to make a comparison. This means that we can only compare variations of the same quantity: length with length, capacity with capacity, etc.;
> Establish the result of this comparison numerically.

In a didactic experiment conducted face-to-face in 2019 by researcher Moya (2020) in the “Educational Work and Schooling” Research Group at the State University of Maringá, it aimed to "[...] promote the organization of teaching based on the principles elaborated in the dialogue between cultural-historical theory, critical-historical pedagogy, and teaching-oriented activity" (MOYA, 2020, p. 184, our translation). It was proposed to the workshop members, among which we included ourselves, a task that sought to identify, from images that refer to the first records of human needs that mobilized the construction of various forms of quantity control, the following aspects: human action, quality, magnitude and its nature (discrete or continuous), a conventional and non-conventional unit of measurement, considering the essential and universal relationship of certain knowledge/action (DAVÍDOV, 1988). Image 4, below, represents one of the moments (particular task) of the mentioned experiment.

**Figure 2 -** Logical-historical movement of quantity control[3]

| Action | Quality | Greatness | Nature | Unconventional/conventional unit of measure | Essential, universal relationship (Davydov 1988, p. 193) |
|---|---|---|---|---|---|
|  1. Sheep Beads | Very/Little | Quantity | Discrete | Unit/Stone | |
|  2. Measure land | Wide/short/short | Length | Continuous | Meter / String | The relationship of multiplicity and divisibility between magnitudes. |
|  3. Measure capacity | Full/empty | Capacity | Continuous | Liters or Kilograms/pots | |

Fonte: Moya (2020, p. 223)

Through this task, we realize, in addition to the three steps previously mentioned, how the concept of magnitude and the other mathematical concepts are at the service of this action of controlling the variation of different magnitudes to enable human beings to live in society. In this sense, the more men appropriate these concepts and mathematical relationships, the

[3]We understand that the title would be more appropriate if, instead of "Logical-historical movement of the control of quantities", it were "Logical-historical movement of different quantities", since it would contemplate the different relations between quantities.

greater the possibility of acting with objects and social phenomena. Because of this, we assert the need for changes in the contents and, necessarily, in the teaching methodologies consolidated in the teaching practice since the child starts school as a cardinal point for developing the scholar's theoretical thinking.

For Davídov (1988, p. 6, our translation), theoretical thinking consists of "[...] a special procedure with which man focuses on the understanding of things and events through the analysis of the conditions of their origin and development. Thus, in the teaching mode proposed and developed by the author, there is a reorganization of school subjects based on a new teaching method based on study tasks, composed of six study actions, which require a variety of specific tasks performed through operations.

The concept of task is understood as the "[...] unity of the objective of the action and the conditions to achieve it" (DAVÍDOV, 1988, p. 178, our translation). Therefore, we understand that the task is linked to the way to obtain the object, that is, to the unity of actions and operations of the activity. For this reason, the members of the OPM study group needed to resume the analysis of the six study actions proposed by Davídov (1988, p. 193) for a better understanding of the teaching of school contents:

> [...] - the transformation of the task data to unveil the universal relation of the studied object;
> - Modeling of the differentiated relation in object form, graphically or using letters;
> - Transformation of the model of the relation to study its properties into "pure form";
> - Construction of the system of particular tasks to be solved by a general procedure;
> - Control of the fulfillment of the previous actions;
> - Evaluation of the assimilation of the general procedure as a result of solving the given study task (DAVÍDOV, 1988, p. 181, our translation).

Initially, the group's biggest challenge was identifying the magnitude of the human actions in each image because, until then, we had not established connections with their particular properties. Therefore, it was necessary to delve into studies of the works of authors such as Davídov (1988, 1982), Caraça (1984), Marx (2008), and Moura *et al.* (2018) to reach the collective synthesis that the concept of magnitude can be defined as a "quality of an object, or phenomenon, that can be quantified", its meaning is not exhausted or determined in an absolute way, but the opposite, it is in movement, since there is "[...] a dialectical movement immanent to things themselves (development of social forms, as the movement of the real, in time, for example)" (FERNANDES, 2008, p. 27, our translation). In this way, we can say that concepts, such as the concept of greatness, are always in motion with the development of social forms permeated by labor relations.

With this experience, it became evident to us, members of GENTEE-OPM, that teaching is structured in articulation with its logical-historical movement, based on humanity's need to produce a certain concept, allowing us to assimilate its particularities using conceptual connections. For Fernandes (2008, p. 27, our translation):

> [...] the chain of thought must begin with what history begins, and its subsequent course must be nothing but the exact image of the historical course in a theoretical and abstract form, but corrected according to the laws given by the actual course of history itself, in which each factor must be considered in the full maturity of its development in its classical form.

When discussing the need for man to act as measuring a piece of land, for example, we realized that it integrates certain particularities that, before we developed the didactic proposal, were fragmented in our thinking, not part of the unitary system of measurement: from the qualities of actions and objects (wide, narrow, long, short), the concept of magnitude (length), its nature (continuous) and the need to establish a standard unit of measurement with the greatest possible accuracy of length (rope, meter, centimeter, etc.).

At this point in the discussion, we moved on to defining the numerical characteristic of a quantity. It became clear to the group that **to measure (quantify), it is necessary to develop the mental action of comparing magnitudes of the same kind, in which one of them is used as the unit of measurement of the other** and can be expressed by a numerical or even geometric representation.

This is not to say that we should follow a linear path for teaching mathematics or that there is a ready-made formula. On the contrary, narrating the history of the "rope tighteners"[4], is unnecessary, for example, to cover the human need to create the concept of measuring. On the contrary, we can work with the logical-historical movement of this concept and put the student in creative tension through a Learning Triggering Situation (LTS), which can be materialized in the form of a pedagogical game of situations emerging from everyday life or as a virtual history of the concept. "These learning triggering situations can propose a problem capable of mobilizing the individual or the collective to solve it" (MOURA; ARAUJO;

[4] “The division of society into classes and private property led to the creation of measures to regulate ownership and the collection of taxes. According to the Greek historian Herodotus, the flooding of the Nile unmarked the boundaries of properties, making it necessary to mark them out. This was done with the help of measurements and plans by the so-called "rope stretchers". Hence the development of fractional numbers. This is mathematics developing in ancient Egypt and Babylon, as it did later with the Mayans and Aztecs” (ROSA NETO, 1988, p. 10, our translation).

SERRÃO, 2018, p. 422, our translation), thus contemplating the teaching objective: the appropriation and development of language at different levels are placed as driving needs for actions towards the object of knowledge, which is also intended to be appropriated, by new generations, as a legitimate cultural heritage.

Understanding the object of study of the subject to be taught is an essential condition for the organization of teaching. In this sense, the logical-historical movement of concept development makes it possible for subjects to become aware of their learning process, provided that education is organized correctly; in the case of mathematics teaching, the relationship between magnitudes as the general foundation for the proper organization of the teaching of this subject.

Because of what was presented, we will delimit mathematical knowledge - which is subdivided into structuring axes of content - to the system of concepts of geometry axis, presenting the main studies that contributed to the composition of our didactic proposal, highlighting the thesis, once assured by Lanner de Moura (1995), that the **human need to control the variations of dimensions [of different quantities] motivated man to measure space**.

By considering the relationship between quantities, the general object of mathematics teaching, we direct ourselves to study its initial genetic connection with the development of human thought, that is, the object-practice activity. Based on what engenders the human thought process, we will find theoretical subsidies to guide us in developing didactic proposals for geometry teaching to address geometric concepts, such as the polygon, through a Learning Triggering Situation (LTS).

In this sense, Davídov (1988, p. 115, our translation) states that "The basis of all human knowledge is the object-practical, productive activity: labor. Also, in line with Marx (2004) and Lenin (1976), the consciousness and universality of man are in practical training, which is broader than theoretical knowledge since, in addition to universality, it holds the immediate reality.

When referring to geometry, Lanner de Moura (1995, p. 54, our emphasis, our translation) adds that

> The first considerations that man makes of geometry originate in simple observations from the human ability to recognize physical configurations and compare shapes and sizes. Numerous life circumstances must have led man to the first geometric elaborations, such as the notion of distance, the need to delimit the land, the construction of walls and dwellings, and others.

> **At the origin of concrete geometric problems with which man is involved since his practical activities are the need to control the variations in dimensions that he faces when defining his physical space to live and produce.**

Controlling the variations in dimensions involves moving object-practical activity such as delimiting land, building walls and dwellings, measuring distance, and others. Lanner de Moura (1995, p. 67, our emphasis, our translation) assures that:

> [...] Just as the need to control the variation of quantities led man to create the number, **the need to control the variations of the dimensions of objects led him to measure space.** This element arises from the relations of men among themselves and with nature when constructing measurement, which we consider as a guiding principle of how to pose the problem of measuring to the child.

Such assumptions lead us to infer that the teaching organization aims to enable the student to perceive the properties and qualities of measurement beyond objects, since the object itself is a complex nexus (VIGOTSKI, 2000). For example, for Davídov (1988), geometry is supported by the practice that exposes and demonstrates, with accuracy, the art of measurement.

Thus, the concept of polygon comprises a particular property in **a system of geometric concepts created by the practical need to control variations in the dimensions of objects and spaces.**

Thus, the teaching organization in this research is subsidized by AOE's theoretical and methodological assumptions for introducing the concept of magnitude, considering the relations major, minor, and equal, through practical objective actions. It is worth noting that:

> Before man, the real concrete appears at first as a sensorial given. Sensory activity in its peculiar forms of contemplation and representation is capable of capturing the integrity of the object, the presence in it of connections that, in the process of knowledge, lead to universality. But contemplation and representation cannot establish the internal character of these connections (DAVÍDOV, 1988, p. 142, our translation).

The author maintains that man can only contemplate and represent the phenomena of reality through a concept through theoretical thought, which allows them to reproduce the system of internal connections that originated the given concrete, discovering its essence. In this way, the teacher's work has concrete thought as its starting and ending point (MARX, 1978).

In this sense, we consider that the genuine connections that created geometric concepts occur through their nuclear relation, that is, the object-practice activity: the human need to control the variations of the dimensions of objects and spaces in the action of measuring, that is what gives them unity.

## System of geometric concepts: The control of the variations of the dimensions of objects and spaces

At this point, we will address how the system of geometric concepts is formed considering the functional place of concepts, especially the idea of the polygon because it is the subject of the study conducted by Ferreira (2022). In identifying the logical-historical movement of the concept of a polygon, we found that it is characterized as a particular property in a system of geometric concepts engendered by the practical need to control the variations of the dimensions of different quantities, whether in land measurements, constructions, or other situations. The challenge at this point concerns the functional place of the polygon in the system of geometric concepts, that is, how it relates to this human need.

Within the framework of the social-historical concept, the "[...] philosophical *category* of *activity* is the theoretical abstraction of the whole universal human practice [...], that is, the collective, adequate, sensory-objective, transforming labor activity of people" (DAVÍDOV, 1988, p. 27, our translation, emphasis added).

Transforming activity of people in the sense that, in the process of collective work, the Higher Psychic Functions (SPF) are developed, among which Vygotsky (2001) highlights the language reflected in the word and expression of thought. For the author, with the development of communality relations, the concept becomes more independent of the word "[...] of the meaning of its expression, resulting in an increasing freedom in its semantic operations and its verbal expression" (VIGOTSKI, 2001 p. 266, our translation). We can cite, as an example, the communality relationship between squares, circles, rectangles, triangles, etc. If we ask ourselves: What do these words or concepts have in common? The answer may reveal that they are plane figures. The terminology plane figures express a whole unitary system of reciprocal relations between variations of two-dimensional geometric figures without the need to discriminate between them.

> If we study the common relation of any concept and its measure of commonality, we will get the surest criterion of the generalization structure of fundamental ideas. To be meaning is the same as to hold itself in certain

> common relations with other meanings, that is, to have a specific measure of commonality.
> Consequently, the nature of the concept, syncretic, in complex, pre-conceptual, is discovered more widely in the specific relations of the concept in question about other concepts (VIGOTSKI, 2001, p. 266, our translation).

Our studies on geometry lead us to understand that the typical relationship between geometric concepts **is ensured by controlling the variations of the dimensions of objects and spaces. Sizes have different generalization structures: one-dimensional (point), one-dimensional (length - lines), two-dimensional (length and width - polygon), and three-dimensional (length, width, and height or depth - polyhedra and non-polyhedra) that vary according to the commonality relations established among geometric concepts in a given degree of application in reality.** The variations in the generalization structure of dimensions are embodied by a specific system of concepts presented in lower and higher conceptual categories.

Thus, through the social need to control the variations of the dimensions of objects and spaces in the action of measuring, the man had to perform "All intellectual operations - comparisons, judgments, conclusions [...]", pointed out by Vygotsky (2001, p. 153, our translation), on three-dimensional objects, in an internal movement of coordinates that determines not only the equivalence of concepts but their measure of generality.

We consider that the control of the variations of object dimensions has been historically and dialectically engendered by the human need to plan a three-dimensional object or space in a particular plane. It is worth noting that "[...] this starting point must maintain its identity throughout the course of reasoning since it constitutes the only guarantee that thought will not get lost on its way" (KOSIK, 1976, p. 29, our translation).

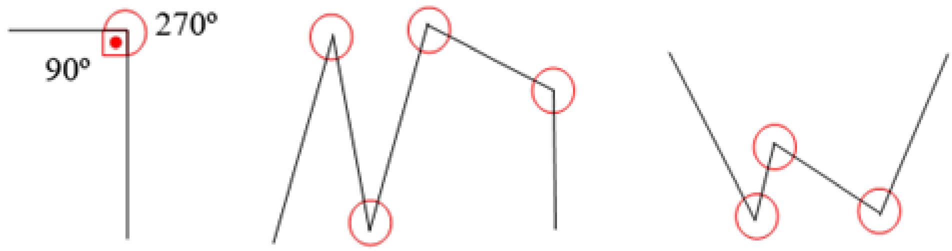
The sensitive and perceptible concrete, constituted by the three-dimensional concept of objects, refers to the starting point still in the chaotic vision of the representation of the whole; thought will reach conceptual abstractions by returning to the concrete, but as concrete thought, articulated and understood (MARX, 2004).

The initial movement that we identify in the planning process of a three-dimensional object refers to the determination of its position in space: the point. Because it does not have volume, length, area, or any other similar dimension, we say that the point is dimensionless. For Wong (2001, p. 42, our translation), "[...] point is the primordial, dimensionless geometric element that indicates location, position. It does not occupy an area or space. It is the beginning and end of a line and is where two lines meet or intersect".

In the preliminary representation of the shape or outline of the object, the plane is dimensioned, the object's initial position with the point, direction, and length with simple lines that follow the same direction, be it horizontal, vertical, or inclined. Or with lines that have a smooth and constant change of direction, opening "out of" the plane (concave) or "into" the plane (convex).

In this process, other common relations can be established and generalized when straight lines become more complex as they meet others with a beginning and end (segment). For example, a sequence of straight line segments that present different directions is called polygonal lines, "polylines", or "broken" lines because when a line segment joins another one, at least two angles are formed, as shown in the following examples.

**Figure 3 -** Forming polygonal lines




Source: Ferreira (2022)

The word polygonal comes from the Greek *polygons*, which means "many angles" because it is formed by *polys*, "many", plus *gonos*, "angles". When we observe the first polygonal line formed by two segments of straight lines, we understand this statement by noting the existence of an angle of 90º and another of 270º. Moreover, it says "broken" lines because they appear broken and "polylines" because they have many lines.

Complex lines can be classified as polygonal, sinuous, or wavy (when they present only curves) and mixed or multilinear (when they have curves and straight lines). In this movement, the dimension of the greatness length unifies and structures these relations. For Vygotsky (2001, p. 263, translation and our emphasis),

> This is a general principle. This is critical to studying the genetic and psychological relations of the general and the particular in children's concepts. Each stage of generalization has its system of relations and commonality. According to the structure of this system, public and specific concepts are distributed in genetic order so that in **the development of concepts, the movement from the general to the particular and from the specific to the general is generally different at each stage of development of meanings,** depending on the structure of generalization that predominates

> at that stage. When you move from one stage to another, the system of communality varies, as does the whole genetic order of development of higher and lower concepts.

We believe that the control of the variations of the dimensions of objects and spaces in the teaching of geometry constitutes the general, unifying concept in which each dimension presents a structure of generalization composed of lower concepts that, according to their development or capture of meanings, are generalized into higher concepts.

In the one-dimensional strokes, when the polygonal (broken) or wavy (curved) lines are closed, they generalize, forming part of a new generalization structure, with a new level of relations in common: the two-dimensional relations with two magnitudes: length and width. Polygonal or wavy closed lines are provided with interior and exterior, perimeter, and area. **The closed polygonal lines, formed by line segments that represent the sides of the figure, are called polygons, which, as we have seen, means many sides or angles.**

A new generalization occurs when we pay attention to the common traits present in the regularities of the sum of the perimeter and the area, the number of segments, and the angles that make up the polygons. In this case, depending on the number of sides and angles, there is a specific shape and nomenclature for the polygon, such as, for example, the square, formed by four equal sides, the same way it happens with the triangle, the rectangle, the rhombus, among others.

An equally sensitive analysis of the nature of these concepts manifests itself more fully when we compare the polygons. For example, the commonality between them expresses an equivalent relationship, and, in different combinations of positions, the congruence of the polygons is established, generalizing them as regular and irregular polygons.

As we know, concepts are not limited to a definition but develop. Similarly, we consider that the two-dimensional figures in geometry are not limited to the concept of the polygon we present. We will only bring the initial connections, however, newly established relations develop new commonalities and concepts with infinite possibilities for generalizations. We limit ourselves to the concept of polygon because it is the concept that is being deepened to think about the organization of geometry teaching. Investigating the development of other geometric concepts would require further study to unveil the interrelationship between the concept systems.

Thus, we will move from the two-dimensional geometric concept system to the three-dimensional one to show that the need for a closer representation of the object in space motivated man to improve his drawing in the plane. With the level of generality already reached with the two-dimensional relations, man can deepen the representation of the object by establishing a new magnitude, the height, composing the three dimensions.

The need to identify the difference in level between one point and another mobilized the individual to seek a reference in the base. The base, already revealed in the two-dimensional relations with the polygons, with the insertion of height or depth (depending on the reference point), enabled man to discover the total dimension of the object, establishing the necessary link to develop a new system that is, a new structure of generalization in the development of geometric concepts.

In this new structure of generalization of thought, other relations of communality begin to take shape with the representation of three-dimensional figures: the die is represented by the cube, the ball, the sphere, the brick, the parallelepiped, etc. New generalizations are made by analyzing the common features between objects or representations of them.

For example: when we observe the representations of objects, we can see that some have two equal polygonal bases and parallel side faces, called prisms. When we examine other representations, we notice that some have only one polygonal base and triangular side faces that meet at a single point, called pyramids. In general, some representations that have all faces, edges, and vertices equal are known as regular polyhedra; others that do not equal all these elements are called irregular polyhedra. The terminology "polyhedron", of Greek origin, means many/many faces. Most polyhedra are designated by the number of faces they have, such as the tetrahedron (4 faces) and hexahedron (6 faces), representing a generalization of three-dimensional figures with flat faces. The same occurs with three-dimensional geometric figures with curved surfaces, known as round bodies.

It is worth pointing out that, as we have already indicated, in this dialectical relationship between geometric concepts, we return to the concrete (objects in space) that was our starting point and that, at this moment, is configured as our point of arrival, understood and articulated as a synthesis of multiple determinations. As Kosik (1976, p. 29, our translation) argued, "From the vital, chaotic, immediate representation of the whole, thought arrives at concepts, at the abstract conceptual determinations, whose formation operates the return to the starting point". In this same direction, Kopnin (1978, p. 162, our translation) states that

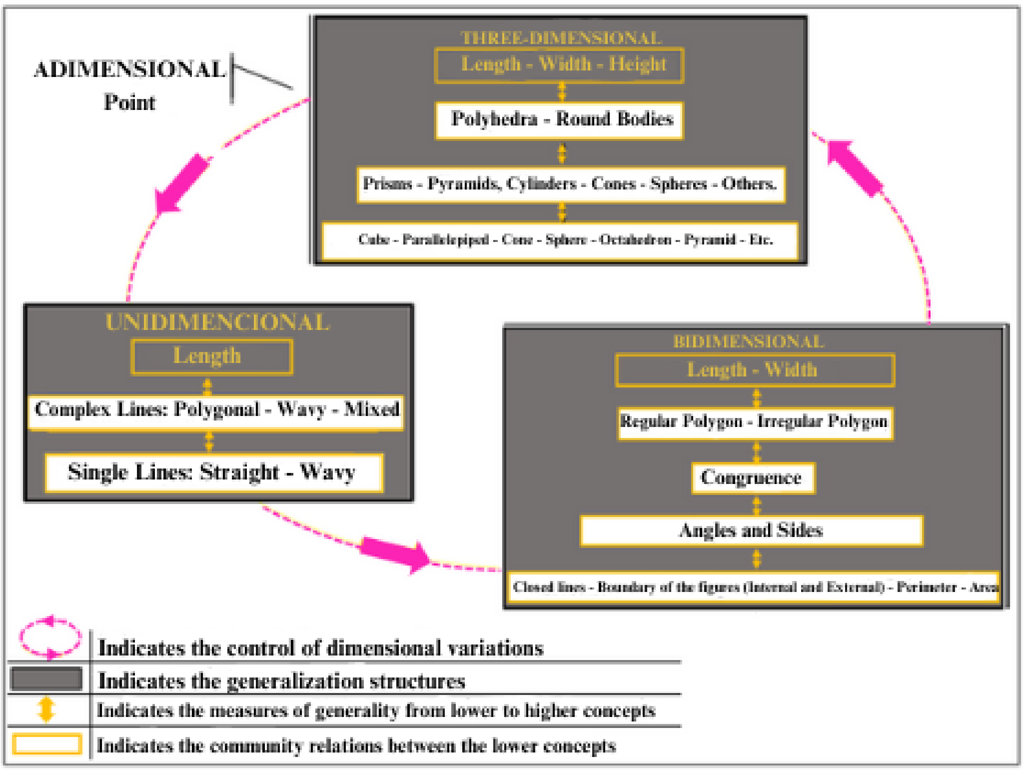
> The movement of knowledge from the sensory-concrete - through the abstract - to the concrete, reproducing the object in the set of abstractions, is

> a manifestation of the law of negation. The abstract is the negation of the sensory-concrete. The concrete in thought negates the abstract, but the mental concrete is not the resumption of the initial, sensory concrete but the result of the ascent to a new, more substantial concrete.

The concrete, in this sense, operates as a result of the ascent to new concrete in its universal expression of the laws of its movement and development. In the internal genetic and dynamic relations of the object within a developing whole, initially, the concrete appears "[...] as the starting point of contemplation and representation, re-elaborated in the concept, and as the mental result of the meeting of abstractions" (DAVIDOV, 1988, p. 150, our translation).

Because of this, in an attempt to expose the initial relations of commonality and generality of the concept of the polygon, we enhanced Figure 2, "Analogy to the relations of generality between concepts," already presented in Ferreira (2017, p. 48, our translation), now evidencing "a" system of geometric concepts and not "the" system of geometric concepts because we understand that this study is not presented finished, but in motion. **We know that the relations among the dimensions give unity to geometry**, in which each size shows a magnitude that triggers the relations of commonality and generality present among the lower concepts that, when generalized into a category hierarchically distinct from the others, become higher. We emphasize that our teaching activity involves the polygon concept, so we focus our reflection on the two-dimensional concept of geometry.

**Figure 4 -** Initial relations of commonality and generality of geometric concepts




Source: Ferreira (2022)

In this rich totality of relations and determinations, we will represent the initial relations of geometric concepts to identify the logical-historical unity of the polygon concept. Finally, we observe that the dimensions of length and width are the essential relations that characterize it as a particular property in the system of geometric concepts, given the practical need to control the variations of the dimensions of objects and spaces in measuring.

## Final considerations

As mentioned, we were challenged to overcome the teaching model that Ferreira (2017) observed through a new organization and materialization of geometry teaching to enable the development of geometric concepts. To do so, we had to understand how a system of concepts is formed to unveil the initial genetic connections of geometric concepts.

Based on one of the assumptions of OAE, the study of the logical-historical movement of creation and development of mathematical concepts toward geometric concepts, that is, their nuclear relation that gives unity to the ideas, we marked the relationship between magnitudes as the general foundation of mathematics teaching, because, in numerical expression, to measure (quantify), it is necessary to develop the mental action of comparing magnitudes of the same quality, where one of them is used as the unit of measurement of the other.

By delimiting this issue to geometry, we identify that its nuclear relation is in the human need to control the variations of the dimensions of objects and spaces in the action of measuring, that is, in the dynamics of the object-practice activity, as in the delimitation of land, construction of walls and houses, distance measurement, among others. We consider that the control of the variations of the dimensions of different magnitudes was engendered (historically and dialectically) by man's need to plan a three-dimensional object or space in a specific plane.

Therefore, by analyzing the constitution of the geometric concept system, we turn to its initial relations, based on the relations of commonality and generality, to walk towards an adequate organization of geometry teaching.

We found that geometric dimensions have different generalization structures: **one-dimensional** (point), **one-dimensional** with length (open lines), **two-dimensional** with length and width (closed lines), and **three-dimensional** with length, width, height, and depth (representation of objects). These structures vary according to the commonality relations between the geometric concepts in a given degree of real application.

Based on this analysis, we highlight the social relations embodied in the labor activity as characteristic of human development, the driving force of the general laws of rationality development and, firstly, of consciousness "[...] as the ability to know things, reaching the concept or essence of them" (CHAUÍ, 2000, p. 45, our translation). For this reason, the relations of the subject and the object with the sociocultural context surrounding them are not exhausted but develop (or recede), intertwined with the dynamics of things themselves, with the collective worldview expressed in ideas, values, and practices of society.

Therefore, we do not intend to exhaust a historical-dialectical analysis of this object but to present contributions that add to the collective effort to promote the teaching of geometry from the perspective of the historical-dialectical materialist method articulated with the general principles of organization of teaching mediated by AOE so that it is a teaching that enables the appropriation of mathematical concepts and the formation of theoretical thinking.

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